

UNSTEADY HEATING IN DE-ICING OF A NONMETALLIC  
PROPELLER BLADE

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An unsteady heat problem is formulated and solved for the heater of an anti-icing device for the nonmetallic blades of an aircraft propeller. Equations are derived of value in the design of such anti-icing devices.

Electric anti-icing devices are at present the most widely used means of preventing the icing of aircraft propellers. They are usually of multi-layer construction with the heating element cemented inside. The outer layer is electrically insulating and protects the heater from mechanical damage. A thermally insulating layer separates the heater from the material of the blade and is intended to reduce the heat flux to the blade.

Let us examine the heat flow kinetics associated with the operation of the heater of an anti-icing device on a nonmetallic blade. Part of the heat will be spent on heating the outer protective layer and removing ice, while the rest will go to heating the blade. The temperature distribution over a section of the blade will then be approximately as illustrated in the figure. The distribution shown does not take account of differences in the thermophysical properties of the separate layers of the section examined. To facilitate solution of the heat problem and to simplify the result, it is convenient to regard the blade, including the layer of ice, as isotropic. It will then be unnecessary to take account of the different thermophysical properties, and equivalent values of the thermal conductivity and diffusivity may be introduced in formulating the problem. The equivalent values  $\lambda_e$  and  $a_e$  will be close to the real ones, since they are approximately of the same order for the materials used in constructing anti-icing devices. An exception is the outer protective layer of steel, but its thickness is small in comparison with the total thickness of the anti-icing device, and its influence on the equivalent thermophysical properties will accordingly be slight.

In addition to this approximation, it may be assumed that the energy losses due to mass transfer do not depend on the temperature of the icing surface. In fact, the vapor pressure of the water over the ice will be a function of temperature. However, we shall assume that, in practice, the vapor pressure of the water over the ice may be taken as the value at the moment when the ice begins to melt, without any appreciable loss of accuracy in the final result. This corresponds to the value at  $-2$  to  $-4^\circ\text{C}$ . This assumption simplifies the solution of the heat problem considerably.

We also assume that the local heat flux is one-dimensional and has a direction at right angles to the blade surface.

These approximations enable one to formulate the unsteady heat problem illustrated in the figure as follows:

$$\frac{\partial t_1(u, Fo)}{\partial Fo} = \frac{\partial^2 t_1(u, Fo)}{\partial u^2} \quad (Fo > 0, 0 < u < 1),$$

$$\frac{\partial t_2(u, Fo)}{\partial Fo} = \frac{\partial^2 t_2(u, Fo)}{\partial u^2} \quad (Fo > 0, 1 < u < \infty).$$
(1)

The boundary conditions will be

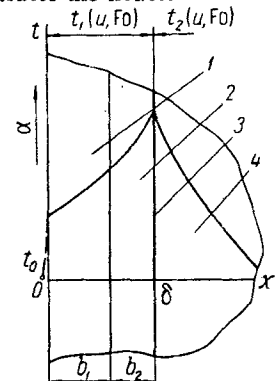
$$t_1(u, 0) = t_2(u, 0) = t_0,$$

$$Bi[t_1(0, Fo) - t_0] + \frac{q_0 \delta}{\lambda_e} - \frac{\partial t_1(0, Fo)}{\partial u} = 0;$$

$$t_1(1, Fo) = t_2(1, Fo),$$

$$-\frac{\partial t_1(1, Fo)}{\partial u} + \frac{\partial t_2(1, Fo)}{\partial u} = \frac{q \delta}{\lambda_e},$$

$$\frac{\partial t_2(\infty, Fo)}{\partial u} = 0.$$
(2)



Unsteady heat conduction in an anti-icing device on a nonmetallic blade:

1) ice, 2) outer layer of anti-icing device, 3) electric heater, 4) propeller blade.

Here

$$q_0 = \frac{\alpha}{c_p} 0.628L_{\text{vap}} \frac{e_0 - e_{t_0}}{\rho_a} \quad (3)$$

Since the difference  $e_0 - e_{t_0}$  is assumed to be constant,  $q_0$  is not a function of time. The equilibrium temperature  $t_0$  of the surface exposed to icing may be determined from the nomograms presented in [2]. The equivalent values  $a_e$  and  $\lambda_e$  are calculated from the formulas:

$$a_e = \frac{\lambda_e}{[c\rho]_e}, \quad [c\rho]_e = \frac{c_{b_1}\rho_{b_1}b_1 + c_1\rho_1l_1 + c_2\rho_2l_2 + \dots + c_n\rho_nl_n}{b_1 + l_1 + l_2 + \dots + l_n},$$

$$\lambda_e = (b_1 + l_1 + l_2 + \dots + l_n) \left( \frac{b_1}{\lambda_{b_1}} + \frac{l_1}{\lambda_1} + \frac{l_2}{\lambda_2} + \dots + \frac{l_n}{\lambda_n} \right)^{-1}.$$

Solving (1), we obtain the following relations:

$$t_1(u, \text{Fo}) - t_0 = \frac{q\delta}{\lambda_e} \sqrt{\text{Fo}} \operatorname{ierfc} \frac{1-u}{2\sqrt{\text{Fo}}} - Q_1 - Q_2, \quad (4)$$

$$t_2(u, \text{Fo}) - t_0 = \frac{q\delta}{\lambda_e} \sqrt{\text{Fo}} \operatorname{ierfc} \frac{u-1}{2\sqrt{\text{Fo}}} - Q_1 - Q_2, \quad (5)$$

where

$$Q_1 = \frac{q\delta}{2\lambda_e \text{Bi}} \left\{ 2 \exp[\text{Bi}^2 \text{Fo} + \text{Bi}(1+u)] \operatorname{erfc} \left( \frac{1+u}{2\sqrt{\text{Fo}}} + \text{Bi} \sqrt{\text{Fo}} \right) + \frac{2\text{Bi} \sqrt{\text{Fo}}}{\sqrt{\pi}} \exp \left[ -\frac{(1+u)^2}{4\text{Fo}} \right] - (2 + \text{Bi} + \text{Bi}u) \operatorname{erfc} \frac{1+u}{2\sqrt{\text{Fo}}} \right\},$$

$$Q_2 = \frac{q_0\delta}{\lambda_e \text{Bi}} \left[ \operatorname{erfc} \left( \frac{u}{2\sqrt{\text{Fo}}} \right) - \exp(\text{Bi}^2 \text{Fo} + \text{Bi}u) \operatorname{erfc} \left( \frac{u}{2\sqrt{\text{Fo}}} + \text{Bi} \sqrt{\text{Fo}} \right) \right],$$

$$\operatorname{ierfc} v = \frac{\exp(-v^2)}{\sqrt{\pi}} - v \operatorname{erfc} v,$$

$$\operatorname{erfc} v = 1 - \operatorname{erf} v = 1 - \frac{2}{\sqrt{\pi}} \int_0^v \exp(-v^2) dv.$$

The thermal power per unit surface area required for de-icing may be determined using the equation for  $t(b_1/\delta, \text{Fo})$  at the time when the ice is shed. The temperature at the ice-blade interface varies with time according to the equation:

$$t \left( \frac{b_1}{\delta}, \text{Fo} \right) - t_0 = \frac{q\delta}{\lambda_e} \left\{ \sqrt{\text{Fo}} \operatorname{ierfc} \left( \frac{k}{2\sqrt{\text{Fo}}} \right) - \frac{1}{\text{Bi}} \exp(\text{Bi}^2 \text{Fo} + 2\text{Bi} - \text{Bi}k) \operatorname{erfc} \left( \frac{2-k}{2\sqrt{\text{Fo}}} + \text{Bi} \sqrt{\text{Fo}} \right) + \frac{\sqrt{\text{Fo}}}{\sqrt{\pi}} \exp \left[ -\frac{(2-k)^2}{4\text{Fo}} \right] - \frac{2+2\text{Bi} - \text{Bi}k}{2\text{Bi}} \operatorname{erfc} \left( \frac{2-k}{2\sqrt{\text{Fo}}} \right) \right\} - \frac{q_0\delta}{\lambda_e \text{Bi}} \left[ \operatorname{erfc} \left( \frac{1-k}{2\sqrt{\text{Fo}}} \right) - \exp(\text{Bi}^2 \text{Fo} + \text{Bi} - \text{Bi}k) \times \operatorname{erfc} \left( \frac{1-k}{2\sqrt{\text{Fo}}} + \text{Bi} \sqrt{\text{Fo}} \right) \right]. \quad (6)$$

If the time at which the heater is switched on is given, we can obviously predetermine the heating power necessary for de-icing. At this power level the temperature at the ice-blade interface at the end of the on-time,  $\text{Fo}_{\text{on}}$ , must be equal to  $0^\circ\text{C}$ . Hence the required thermal power is determined from the equation

$$q = \left\{ \frac{q_0}{\text{Bi}} \left[ \operatorname{erfc} \left( \frac{1-k}{2\sqrt{\text{Fo}}} \right) - \exp(\text{Bi}^2 \text{Fo} + \text{Bi} - \text{Bi}k) \times \right. \right. \quad (7)$$

$$\begin{aligned}
& \times \operatorname{erfc}\left(\frac{1-k}{2\sqrt{Fo}} + Bi\sqrt{Fo}\right) - \frac{\lambda_e}{\delta} t_0 \left\{ \sqrt{Fo} \operatorname{ierfc}\left(\frac{k}{2\sqrt{Fo}}\right) - \right. \\
& \left. - \frac{2+2Bi-Bi k}{Bi} \operatorname{erfc}\left(\frac{2-k}{2\sqrt{Fo}}\right) + \frac{\sqrt{Fo}}{\sqrt{\pi}} \exp\left[-\frac{(2-k)^2}{4Fo}\right] - \right. \\
& \left. - \frac{1}{Bi} \exp(Bi^2Fo + 2Bi - Bi k) \operatorname{erfc}\left(\frac{2-k}{2\sqrt{Fo}} + Bi\sqrt{Fo}\right) \right\}^{-1}. \quad (7)
\end{aligned}$$

(cont'd)

Note that a certain time is needed for the ice to melt, and so the thermal power obtained from (7) will be somewhat below that actually required. We may determine the additional power necessary to melt the ice from the formula

$$Q_m = \rho_i \Delta b L_m. \quad (8)$$

The heat flux given by (7) is then increased by  $Q_m$ .

At the end of the on-time the temperature of a heater with thermal power  $q$  will be described by the expression

$$\begin{aligned}
t(1, Fo) = t_0 + \frac{q\delta}{\lambda_e} \left\{ 0,5642 \sqrt{Fo} - \frac{1}{Bi} \left[ \exp(Bi^2Fo + 2Bi) \times \right. \right. \\
\left. \times \operatorname{erfc}\left(\frac{1}{\sqrt{Fo}} + Bi\sqrt{Fo}\right) - \frac{Bi\sqrt{Fo}}{\sqrt{\pi}} \exp\left(-\frac{1}{Fo}\right) - \frac{1+Bi}{Bi} \operatorname{erfc}\left(\frac{1}{\sqrt{Fo}}\right) \right] - \right. \\
\left. - \frac{a_0}{q Bi} \left[ \operatorname{erfc}\left(\frac{1}{2\sqrt{Fo}}\right) - \exp(Bi^2Fo + Bi) \operatorname{erfc}\left(\frac{1}{2\sqrt{Fo}} + Bi\sqrt{Fo}\right) \right] \right\}. \quad (9)
\end{aligned}$$

#### NOTATION

$u = x/\delta$  - relative thickness coordinate;  $x$  - thickness coordinate;  $\delta$  - fixed value of  $x$ ;  $Fo = d_e\tau/\delta^2$  - homochronous (Fourier) number;  $d_e$  - equivalent thermal diffusivity;  $\tau$  - time;  $t(u, Fo)$  - temperature as a function of  $u$  and  $Fo$ ;  $b_1$  - thickness of ice layer;  $b_2$  - thickness of outer layer of anti-icing device;  $Bi = \alpha\delta/\lambda_e$  - Biot number;  $\alpha$  - convective heat transfer coefficient;  $\lambda_e$  - equivalent thermal conductivity;  $t_0$  - equilibrium temperature of icing surface of propeller blade;  $l_1, l_2, l_3$ , etc., - respective thicknesses of layers of materials of which the anti-icing device is made;  $[\rho]_e$  - equivalent volumetric heat capacity;  $q$  - heat flux (specific surface heat power of electric heater);  $c_p$  - specific heat of air at constant pressure;  $L_{vap}$  - heat of vaporization of water at  $t_0$ ;  $e_0 - t_0$  - difference of vapor pressures of water over icing surface at  $-2$  to  $-4^\circ\text{C}$  and in medium surrounding blade;  $p_a$  - atmospheric pressure at flight altitude;  $k = b_2/\delta$  - relative thickness;  $\rho_i$  - density of ice;  $\Delta b$  - thickness of layer when ice melts and is shed (0.05-0.10 mm);  $L_m$  - heat of fusion of ice.

#### REFERENCES

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